
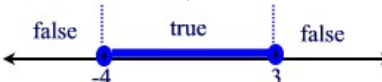


Algebra 2 – Things to Remember!



<p>Exponents:</p> $x^0 = 1$ $x^m \cdot x^n = x^{m+n}$ $\frac{x^m}{x^n} = x^{m-n}$ $(xy)^n = x^n \cdot y^n$ $x^{-m} = \frac{1}{x^m}$ $(x^n)^m = x^{n \cdot m}$ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	<p>Complex Numbers:</p> $\sqrt{-1} = i \quad \sqrt{-a} = i\sqrt{a}; a \geq 0$ $i^2 = -1 \quad i^4 = i^2 = -1 \text{ divide exponent by 4, use remainder, solve.}$ <p>$(a + bi)$ conjugate $(a - bi)$</p> $(a + bi)(a - bi) = a^2 + b^2$ $ a + bi = \sqrt{a^2 + b^2} \text{ absolute value=magnitude}$	<p>Logarithms</p> $y = \log_b x \Leftrightarrow x = b^y$ <p>$\ln x = \log_e x$ natural log $e = 2.71828\dots$ $\log x = \log_{10} x$ common log</p> <p>Change of base formula: $\log_b a = \frac{\log a}{\log b}$</p> <p>Properties of Logs:</p> $\log_b b = 1 \quad \log_b 1 = 0$ $\log_b (m \cdot n) = \log_b m + \log_b n$ $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$ $\log_b (m^r) = r \log_b m$ <p>Domain: $\log_b x$ is $x > 0$</p>
<p>Factoring:</p> <p>Look to see if there is a GCF (greatest common factor) first. $ab + ac = a(b + c)$</p> $x^2 - a^2 = (x - a)(x + a)$ $(x + a)^2 = x^2 + 2ax + a^2$ $(x - a)^2 = x^2 - 2ax + a^2$ <p>Factor by Grouping:</p>  $x^3 + 2x^2 - 3x - 6$ <p>$(x^3 + 2x^2) - (3x + 6)$ group</p> <p>$x^2(x + 2) - 3(x + 2)$ factor each</p> <p>$(x^2 - 3)(x + 2)$ factor</p>	<p>Exponentials $e^x = \exp(x)$</p> $b^x = b^y \rightarrow x = y \quad (b > 0 \text{ and } b \neq 1)$ <p>If the bases are the same, set the exponents equal and solve.</p> <p>Solving exponential equations:</p> <ol style="list-style-type: none"> 1. Isolate exponential expression. 2. Take \log or \ln of both sides. 3. Solve for the variable. <p>$\ln(x)$ and e^x are inverse functions</p> $\ln e^x = x \quad e^{\ln x} = x$ $\ln e = 1 \quad e^{\ln 4} = 4$ $e^{2 \ln 3} = e^{\ln 3^2} = 9$	<p>Quadratic Equations: $ax^2 + bx + c = 0$ (Set = 0.)</p> <p>Solve by factoring, completing the square, quadratic formula.</p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p>$b^2 - 4ac > 0$ two real unequal roots</p> <p>$b^2 - 4ac = 0$ repeated real roots</p> <p>$b^2 - 4ac < 0$ two complex roots</p> <p>Square root property: If $x^2 = m$, then $x = \pm\sqrt{m}$</p> <p>Completing the square: $x^2 - 2x - 5 = 0$</p> <ol style="list-style-type: none"> 1. If other than one, divide by coefficient of x^2 2. Move constant term to other side $x^2 - 2x = 5$ 3. Take half of coefficient of x, square it, add to both sides $x^2 - 2x + \boxed{1} = 5 + \boxed{1}$ <ol style="list-style-type: none"> 4. Factor perfect square on left side. $(x - 1)^2 = 6$ 5. Use square root property to solve and get two answers. $x = 1 \pm \sqrt{6}$
<p>Variation: always involves the constant of proportionality, k. Find k, and then proceed.</p> <p>Direct variation: $y = kx$</p> <p>Inverse variation: $y = \frac{k}{x}$</p> <p>Varies jointly: $y = kxj$</p> <p>Combo: Sales vary directly with advertising and inversely with candy cost.</p> $y = \frac{ka}{c}$	<p>Absolute Value: $a > 0$</p> $ a = \begin{cases} a; & a \geq 0 \\ -a; & a < 0 \end{cases}$ $ m = b \Rightarrow m = -b \text{ or } m = b$ $ m < b \Rightarrow -b < m < b$ $ m > b \Rightarrow b > m \text{ or } b < m$	<p>Sum of roots: $r_1 + r_2 = -\frac{b}{a}$ Product of roots: $r_1 \cdot r_2 = \frac{c}{a}$</p> <p>Inequalities: $x^2 + x - 12 \leq 0$ Change to =, factor, locate critical points on number line, check each section.</p> $(x + 4)(x - 3) = 0$ $x = -4; x = 3$  <p>ANSWER: $-4 \leq x \leq 3$ or $[-4, 3]$ (in interval notation)</p>

Radicals: Remember to use fractional exponents.

$$\sqrt[n]{x} = x^{\frac{1}{n}} \quad x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$\sqrt[n]{a^n} = a \quad \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Simplify: look for perfect powers.

$$\sqrt{x^{12}y^{17}} = \sqrt{x^{12}y^{16}y} = x^6y^8\sqrt{y}$$

$$\sqrt[3]{72x^9y^8z^3} = \sqrt[3]{8 \cdot 9x^8xy^8z^3} = 2x^2y^2z\sqrt[3]{9x}$$

Use conjugates to rationalize denominators:

$$\frac{5}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{10-5\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-\sqrt{9}} = 10-5\sqrt{3}$$

Equations: isolate the radical; square both sides to eliminate radical; combine; solve.

$$2x-5\sqrt{x}-3=0 \rightarrow (2x-3)^2 = (5\sqrt{x})^2$$

$$4x^2-12x+9=25x \rightarrow \text{solve: } x=9; x=1/4$$

CHECK ANSWERS. Answer only $x = 9$.

Functions: A function is a set of ordered pairs in which each x -element has only ONE y -element associated with it.

Vertical Line Test: is this graph a function?

Domain: x -values used; **Range:** y -values used

Onto: all elements in B used.

1-to-1: no element in B used more than once.

Composition: $(f \circ g)(x) = f(g(x))$

Inverse functions f & g : $f(g(x)) = g(f(x)) = x$

Horizontal line test: will inverse be a function?

Transformations:

$-f(x)$ over x -axis; $f(-x)$ over y -axis

$f(x+a)$ horizontal shift; $f(x)+a$ vertical shift

$f(ax)$ stretch horizontal; $af(x)$ stretch vertical

Working with Rationals (Fractions):

Simplify:

remember to look for a factoring of -1 :

$$\frac{3x-1}{1-3x} = \frac{-1(-3x+1)}{1-3x} = -1$$

Add: Get the common denominator.

Factor first if possible:

Multiply and Divide: Factor First

Rational Inequalities

$$\frac{x^2-3x-15}{x-2} \geq 0 \text{ The critical values}$$

from factoring the numerator are $-3, 5$.

The denominator is zero at $x = 2$.

Place on number line, and test sections.



Sequences

Arithmetic: $a_n = a_1 + (n-1)d$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Geometric: $a_n = a_1 \cdot r^{n-1}$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Recursive: Example:

$$a_1 = 4; \quad a_n = 2a_{n-1}$$

Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

Solving Rational Equations:

Get rid of the denominators by mult. all terms by common denominator.

$$\frac{22}{2x^2-9x-5} - \frac{3}{2x+1} = \frac{2}{x-5}$$

multiply all by $2x^2-9x-5$ and get

$$22-3(x-5) = 2(2x+1)$$

$$22-3x+15 = 4x+2$$

$$37-3x = 4x+2$$

$$35 = 7x$$

$$5 = x$$

Great! But the only problem is that

$x = 5$ does not CHECK!!!! There is no solution.

Extraneous root.

Motto: Always CHECK ANSWERS.

Equations of Circles: $x^2 + y^2 = r^2$ center origin

$(x-h)^2 + (y-k)^2 = r^2$ center at (h,k)

$x^2 + y^2 + Cx + Dy + E = 0$ standard form

Complex Fractions:

Remember that the fraction bar means divide:

Method 1: Get common denominator top and bottom

$$\frac{\frac{2}{x^2} - \frac{4}{x}}{\frac{4}{x} - \frac{2}{x^2}} = \frac{\frac{2-4x}{x^2}}{\frac{4x-2}{x^2}} = \frac{2-4x}{x^2} \div \frac{4x-2}{x^2} = \frac{2-4x}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{4x-2} = -1$$

Method 2: Mult. all terms by common denominator for all.

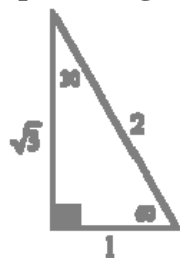
$$\frac{\frac{2}{x^2} - \frac{4}{x}}{\frac{4}{x} - \frac{2}{x^2}} = \frac{x^2 \cdot \frac{2}{x^2} - x^2 \cdot \frac{4}{x}}{x^2 \cdot \frac{4}{x} - x^2 \cdot \frac{2}{x^2}} = \frac{2-4x}{4x-2} = -1$$

Trigonometry – Things to Remember!

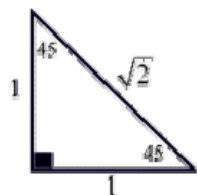


Arc Length of a Circle = θr (in radians)

Special Right Triangles



30°-60°-90° triangle
side opposite 30° = 1/2 hypotenuse
side opposite 60° = 1/2 hypotenuse $\sqrt{3}$



45°-45°-90° triangle
hypotenuse = leg $\sqrt{2}$
leg = 1/2 hypotenuse $\sqrt{2}$

Law of Sines: uses 2 sides and 2 angles
 $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ Has an ambiguous case.

Law of Cosines: uses 3 sides and 1 angle
 $c^2 = a^2 + b^2 - 2ab \cos C$

Area of triangle: $A = \frac{1}{2} ab \sin C$
Area of parallelogram: $A = ab \sin C$

Pythagorean Identities:
 $\sin^2 \theta + \cos^2 \theta = 1$ $\tan^2 \theta + 1 = \sec^2 \theta$
 $1 + \cot^2 \theta = \csc^2 \theta$

Radians and Degrees

Change to radians multiply by $\frac{\pi}{180}$

Change to degrees multiply by $\frac{180}{\pi}$

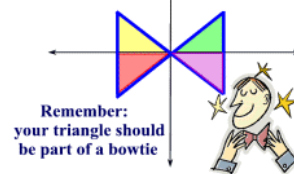
Quadrantal angles – 0, 90, 180, 270

CoFunctions: examples
 $\sin \theta = \cos(90^\circ - \theta)$; $\tan \theta = \cot(90^\circ - \theta)$

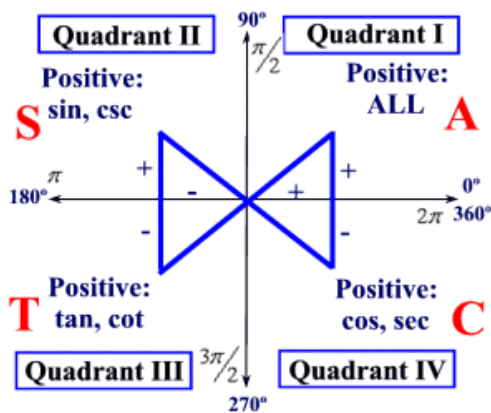
Inverse notation:

$\arcsin(x) = \sin^{-1}(x)$
 $\arccos(x) = \cos^{-1}(x)$
 $\arctan(x) = \tan^{-1}(x)$

Reference triangles are drawn to the x-axis.



Remember: your triangle should be part of a bowtie



Trig Functions

$\sin \theta = \frac{o}{h}$; $\cos \theta = \frac{a}{h}$; $\tan \theta = \frac{o}{a}$

$\csc \theta = \frac{h}{o}$; $\sec \theta = \frac{h}{a}$; $\cot \theta = \frac{a}{o}$

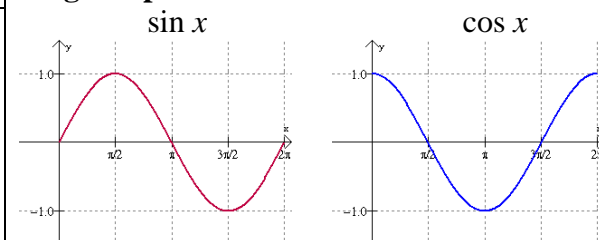
Reciprocal Functions

$\sin \theta = \frac{1}{\csc \theta}$; $\cos \theta = \frac{1}{\sec \theta}$; $\tan \theta = \frac{1}{\cot \theta}$

$\csc \theta = \frac{1}{\sin \theta}$; $\sec \theta = \frac{1}{\cos \theta}$; $\cot \theta = \frac{1}{\tan \theta}$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$

Trig Graphs



amplitude = $\frac{1}{2} | \max - \min |$ (think height)

period = horizontal length of 1 complete cycle

frequency = number of cycles in 2π

sinusoidal curve = any curve expressed as
 $y = A \sin(B(x - C)) + D$

phase shift = measure of horizontal shifting

Statistics and Probability – Things to Remember!

Statistics:

$$\text{mean} = \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

median = middle number in ordered data

mode = value occurring most often

range = difference between largest and smallest

mean absolute deviation (MAD):

$$\text{population MAD} = \frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|$$

variance:

$$\text{population variance} = (\sigma x)^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

standard deviation:

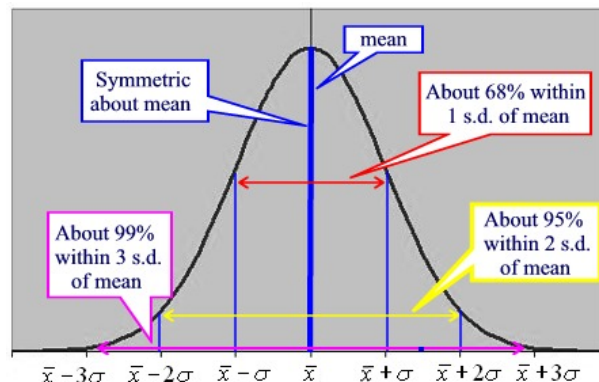
population standard deviation =

$$\sigma x = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

S_x = sample standard deviation

σ_x = population standard deviation

Normal Distribution and Standard Deviation



Binomial Probability

$${}_n C_r \cdot p^r \cdot q^{n-r} \text{ exactly } r \text{ times}$$

$$\text{or } \binom{n}{r} \cdot p^r \cdot (1-p)^{n-r}$$

When computing "**at least**" and "**at most**" probabilities, it is necessary to consider, in addition to the given probability,

- all probabilities larger than the given probability ("at least")
- all probabilities smaller than the given probability ("at most")

Probability

Permutation: without replacement and order matters

$${}_n P_r = \frac{n!}{(n-r)!}$$

Combination: without replacement and order does not matter

$${}_n C_r = \binom{n}{r} = \frac{{}_n P_r}{r!} = \frac{n!}{r!(n-r)!}$$

Empirical Probability

$$P(E) = \frac{\text{\# of times event } E \text{ occurs}}{\text{total \# of observed occurrences}}$$

Theoretical Probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{\# of outcomes in } E}{\text{total \# of outcomes in } S}$$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$P(A') = 1 - P(A)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

for not mutually exclusive

$$P(A \text{ or } B) = P(A) + P(B)$$

for mutually exclusive

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)} \text{ (conditional)}$$